An Investigation of Permanence and Exclusion in a Two-Dimensional Discrete Time Competing Species Model

by
Patrick J. Johnson, B.Sc., M.Sc.

A thesis submitted for the award of Ph. D.

Department of Mathematics and Statistics,
University *of* Limerick

Supervisor: Dr. Mark E. Burke

Submitted to the University *of* Limerick, Ireland, June 2006

Extended Abstract

In this work, a discrete version of the Lotka-Volterra equations (4) was used to model competition between two species. Denoting the populations of the two species at time n by x_n and y_n respectively, it is possible to derive a discrete time two-dimensional quadratic system of equations represented by the map $T : R^2_+ \to R^2_+$ where $T(x_n, y_n) = (x_{n+1}, y_{n+1})$ with

$$x_{n+1} = ax_n(1 - x_n - sy_n),$$
$$y_{n+1} = by_n(1 - y_n - tx_n),$$

where a and b represent the growth rates of the species and s and t are the interspecific competition parameters. When s and t are both less than 1 the self-limiting terms of the map dominate the eventual evolution of the system. In contrast to this, when s and t are both greater than 1, competition between the species plays a more important role and thus has a more significant effect on the overall evolution of the system. The *mixed* case of one competition parameter greater than 1 and the other less than 1 is also possible. Depending on which situation is being examined it is possible for either the coexistence of both species or the extinction of one or other of the competing species to occur.

It is obvious that several outcomes are possible in this type of system but in general these outcomes can be classified under the group headings of "permanence" (2) and "competitive exclusion" (1). Permanence occurs within a two species system when it is possible for both the species to stably coexist. Competitive exclusion on the other hand occurs within a competing species system when one of the species is driven to extinction.

A significant problem arising in this system is that not all initial trajectories starting in R^2_+ are *feasible trajectories* of the system. An initial point in R^2_+ is said to have a feasible trajectory if the initial point and all future iterates of the point, under the map T, remain nonnegative in both x and y. Knowing whether or not there exists a *feasible region* and being able to calculate the structure of this region are vital prerequisites to any discussion of permanence and exclusion in the system.

The work of this thesis is concerned with investigating when feasible trajectories exist and under what conditions the phenomena of permanence and competitive exclusion are exhibited in the above discrete time system. The analysis of the system is carried out using the Method of Critical Curves as well as traditional methods in the fields of nonlinear dynamics and population dynamics. Since its introduction, the Method of Critical Curves has proven to be an important tool in the analysis of the global dynamical properties of noninvertible maps (3).

References

[1] Armstrong R, McGehee R. Competitive exclusion. The American Naturalist 1980;115(2):151-170.

[2] Hofbauer J, Hutson V, Jansen W. Coexistance for systems governed by difference equations of Lotka-Volterra type. Journal of Mathematical Biology 1987;25:553-570.

[3] Mira, C., Gardini, L., Barugola, A., Cathala, J.C.: Chaotic dynamics in two-dimensional noninvertible maps. World Scientific, Singapore (1996)

[4] Volterra V. Variations and fluctuations of the number of individuals in animal species living together. Journal du Conseil International pour l'Exploration de la Mer 1928;3:3-51.