Graphical Methods for a 1st order Flow

The 1-d flow $\dot{x} = f(x)$ has a fixed point (equilibrium) at x_e where x_e is the solution of $0 = f(x_e)$.

Recall that the sign of $\dot{x}(= \frac{dx}{dt})$ tells us whether x is increasing or decreasing as a function of t. Consider the plot of $f(x)$ v. x.

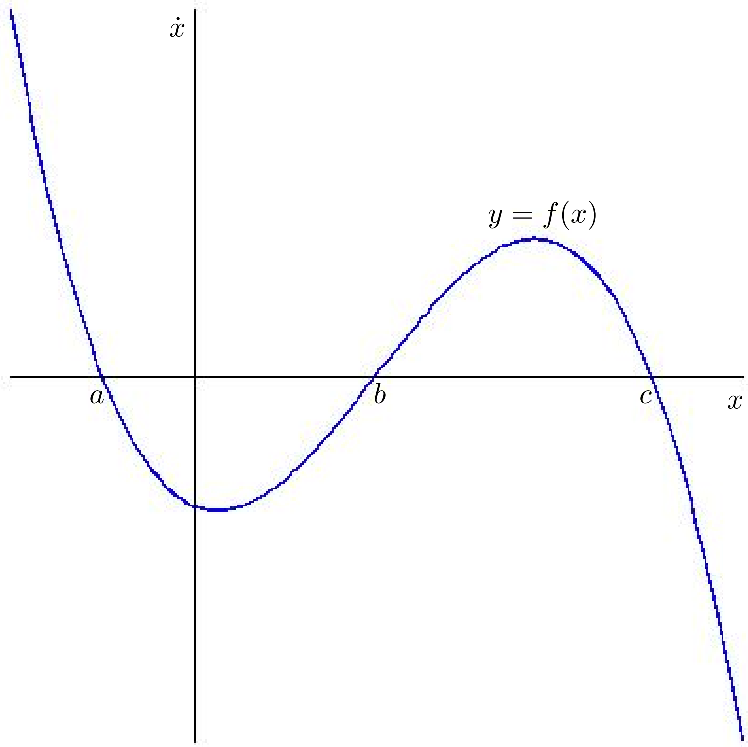

Figure 1: Determining stability of 1-d flow: $f(x)$ v. x.

The fixed points are given by the solutions of $f(x) = 0$ - thus in Fig. 1 for example $x_e = a$, b and c. If $f(x) > 0$ then $x(t)$ is increasing ($\uparrow$), while if $f(x) < 0$ then $x(t)$ is decreasing ($\downarrow$). Just to the left of $x_e = a$, $f(x) > 0$ and so $x(t) \uparrow a$. Just to the right of $x_e = a$, $f(x) < 0$ and so $x(t) \downarrow a$. We conclude that $x_e = a$ is locally stable.
A similar style of argument leads to the conclusion that $x_e = b$ is locally unstable and $x_e = c$ is locally stable.

If f is differentiable at x_e (as is the case at all of the fixed points in Fig. 1) then

- If $Df(x_e) < 0$, then x_e is locally stable.
- If $Df(x_e) > 0$, then x_e is locally unstable.

Exercise: What are the stability properties of the fixed points in Fig. 2 ?

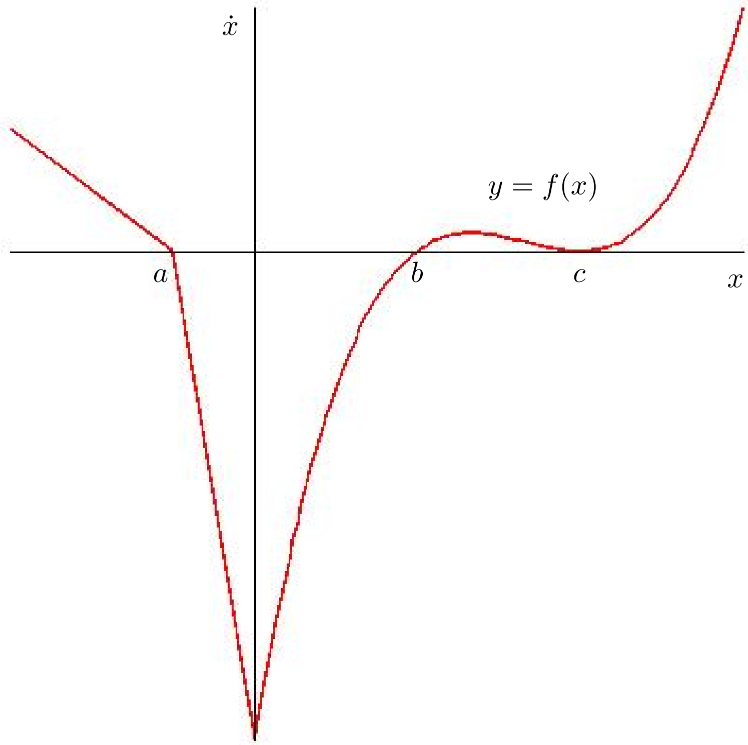

Figure 2: Which fixed points are stable? Non-differentiable 1-d flow : $f(x)$ v. x.

Based on Fig. 2, does the term *semi-stable* makes sense?