Implementing State Feedback with State Estimators

For the continuous-time LTI CC & CO system

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u} & (1) \\ \mathbf{y} &= C\mathbf{x} & (2) \end{aligned}$$

with full state estimator

$$\dot{\mathbf{z}} = (A + LC)\mathbf{z} - L\mathbf{y} + B\mathbf{u} \quad (3)$$

we feedback the observer output to implement the control:

$$\mathbf{u} = K\mathbf{z} + \mathbf{v} \quad (4)$$

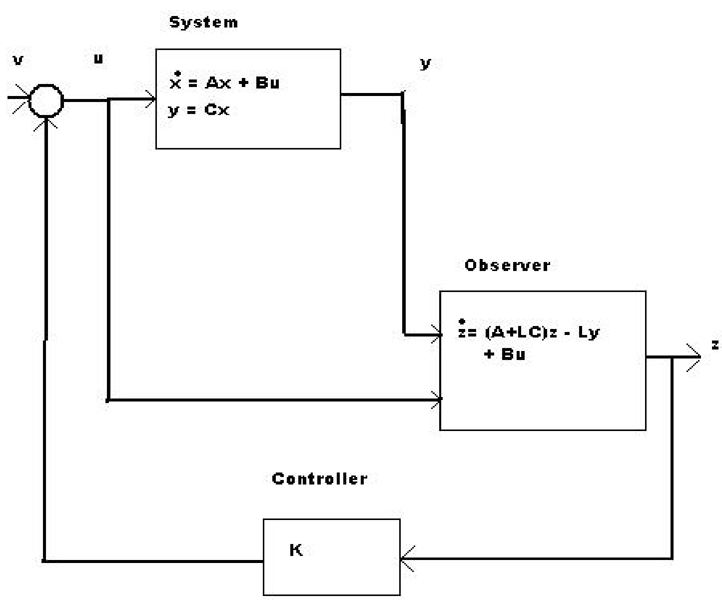

Figure 1: Linear Feedback via Observer

The "super-system" consisting of the original system plus observer has order $2n$ and is governed by the equations got by substituting from Eqs (2) & (4) into Eqs (1) & (3):

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + BK\mathbf{z} + B\mathbf{v} \\ \dot{\mathbf{z}} &= (A + LC + BK)\mathbf{z} - LC\mathbf{x} + B\mathbf{v} \end{aligned}$$

or in vector-matrix formulation

$$\frac{d}{dt}\begin{pmatrix} \mathbf{x} \\ \mathbf{z} \end{pmatrix} = \begin{pmatrix} A & BK \\ -LC & A+LC+BK \end{pmatrix}\begin{pmatrix} \mathbf{x} \\ \mathbf{z} \end{pmatrix} + \begin{pmatrix} B \\ B \end{pmatrix}\mathbf{v} \tag{5}$$

What are the poles of this super-system? Using the change of variable

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{z} \end{pmatrix} = \begin{pmatrix} I & -I \\ I & 0 \end{pmatrix}\begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{z}} \end{pmatrix}$$

the super-system state equation becomes

$$\frac{d}{dt}\begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{z}} \end{pmatrix} = \begin{pmatrix} A+BK & LC \\ 0 & A+LC \end{pmatrix}\begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{z}} \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix}\mathbf{v} \tag{6}$$

The new state matrix is block-triangular and thus its eigenvalues (C-L poles) are those of the diagonal blocks $A + BK$ and $A + LC$, which are the eigenvalues of (i) the original system with state feedback and (ii) the observer respectively. Hence as $t \to \infty$, the behaviour got by feeding back $\mathbf{z}$ will be the same as that which would have resulted from feeding back $\mathbf{x}$ had it been available.

Food for thought: Is the super-system CC? (work with the transformed system).