Let $\{a_n\}$ represent a sequence of numbers.

1. If $a_n = 2n^2 + 2n + 1$ write down:

   (a) a_1, a_2, a_6

   (b) $a_{n+1}, a_{2n}, a_{n+4}, a_k, a_n + 4$

   Show that a_n satisfies the recurrence relation

   $$a_{n+2} = 2a_{n+1} - a_n + 4,$$

2. If $a_n = n!$ ("factorial n"), write down:

   $$a_1, a_3, a_6$$

   Can you find a recurrence relationship between a_{n+1} and a_n ? [Recall that $n! = n(n-1)(n-2)\ldots 3 \times 2 \times 1$ if $n > 0$ and $0! = 1$ (by definition)].

3. Which if any of the following sequences are (i) arithmetic, (ii) geometric, (iii) fibonacci-like ? Can you find recurrence relations for each the sequences? Find the general term (a_n) for each sequence.

   (a) $1, 3, 5, 7, 9, \ldots$

   (b) $1, 4, 9, 16, 25, \ldots$

   (c) $2, 5, 11, 23, 47, \ldots$

   (d) $-1, 1, 0, 1, 1, 2, 3, 5, 8, \ldots$

   (e) $1, 2, 24, 720, 40320, \ldots$

   (f) $2, 1, 2, 5, 10, 17, \ldots$ Hint: Consider a new sequence got by subtracting the previous term from the current term ($a_n - a_{n-1}$). Does this help? You might need to try this approach more than once.

   (g) $2, 3, 6, 18, 108, 1944, \ldots$ Hint: consider getting the prime factorisation of each term in the sequence; do you recognise the pattern?

   (h) $1, 11, 21, 1211, 111221, 312211, \ldots$ [1]

4. What is the next term in the sequence $1, 2, 3, \ldots$?
   4 say all of us! However the mathematical genii tells me that the general term is

   $$a_n = \frac{n^3 - 6n^2 + 17n - 6}{6}$$

   from which we calculate that $a_4 = 5$. [Check that the formula gives $a_1 = 1, a_2 = 2$ and $a_3 = 3$.] Ok, we can't win them all.
   In point of fact, no matter what number is chosen for a_4 ($= x$, say), there is a formula along the lines of the one above which generates the sequence beginning $1, 2, 3, x, \ldots$.

[1] When you've stopped hitting your head off the wall, google on "look and say" sequence.