UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2007

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: **Full marks for correct answers to 5 questions.**
A table of *Laplace* transforms is attached.

1. (a) Sketch the following functions and find their *Laplace* transforms:

(i) $f(t) = \begin{cases} 0, & \text{if } t < 3 \\ \cosh(t-3), & \text{if } t \geq 3 \end{cases}$ 2

(ii) $f(t) = \begin{cases} \sin \pi t, & \text{if } 0 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$ 3

(iii) $f(t) = \begin{cases} 0, & \text{if } t < 0 \\ t, & \text{if } 0 \leq t < 1 \\ f(t-1), & \text{if } t \geq 1 \end{cases}$ 3

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$$(i)\ \frac{s+2}{s^2-3s+2} \qquad (ii)\ \frac{e^{-3s}}{s^2+2s} \qquad (iii)\ \ln\frac{s^2}{s+1}$$

2. (a) A mass of 1 kg moving in the horizontal plane under the influence of an externally applied force (of $f(t)$ N at time t.) is connected by means of a spring to a wall and is subject to damping. If the spring constant is 4 N/m and the damping coefficient is 5 N-s/m, show that $y(t)$, the displacement of the mass from its equilibrium position satisfies the differential equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = f(t)$$

4

(b) Use *Laplace* transforms to solve the initial value problem 12

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = f(t), \qquad y(0) = 0, \quad \frac{dy}{dt}(0) = 0$$

where

$$f(t) = \begin{cases} 4, & \text{if } 0 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

3. (a) Find the *Fourier* series of the periodic function 6

$$f(x) = \begin{cases} \pi + x, & \text{if } -\pi < x \leq 0 \\ -\pi + x, & \text{if } 0 < x \leq \pi \end{cases} \qquad f(x+2\pi) = f(x)$$

(b) Hence evaluate the series 4

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(c) Use the *Fourier* series of part (a) to find a particular solution of the differential equation 6

$$\frac{d^2y}{dx^2} + 4y = f(x)$$

where $f(x)$ is as in part (a).

4. (a) Let P_2 be the space of polynomials of degree at most two, i.e. $P_2 = \{p(x) : p(x) = a_0 + a_1x + a_2x^2\}$ where $a_0,\ a_1$ and a_2 are real numbers. Determine whether the set of polynomials

$$\{1 - 2x,\ 2 + x^2,\ -1 + 4x + \frac{x^2}{2}\}$$

forms a basis for P_2 8

(b) Determine whether the following set of vectors in $\mathbf{R^3}$ is linearly independent. 8

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \right\}$$

5. (a) Let P_1 be the space of polynomials of degree at most one. Use the *Gram-Schmidt* process to transform the standard basis $\{1,\ x\}$ for P_1 to an orthonormal one defined by the inner product

$$< p, q >= \sum_{i=1}^{5} p(x_i)q(x_i)$$

where $x_1 = 0,\ x_2 = 1,\ x_3 = 2,\ x_4 = 3,\ x_5 = 4.$ 8

(b) Let f be a function on $[0, 4]$, taking the value $f(x_i)$ at $x = x_i,\ i = 1, 2, 3, 4, 5$ as given in the following table

x_i	$f(x_i)$
0	-2
1	1
2	2
3	6
4	8

Find the least squares approximation to f in P_1 using the inner product defined in part (a). 8

6. Find the eigenvalues and corresponding eigenspaces of the matrix 8

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & 1 & -3 \end{pmatrix}$$

Is A diagonalisable over the reals ? If so, find the matrix P which diagonalises it. 8

7. (a) Find the *Cholesky* decomposition ($A = LL^T$) of the matrix 5

$$A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 5 & -4 \\ 1 & -4 & 6 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where 5

$$b = \begin{pmatrix} -1 \\ 6 \\ -11 \end{pmatrix}$$

(b) Define the condition number of a square matrix. Of what is it a measure ? 2

For the matrix A of part (a) verify that its inverse is given by

$$A^{-1} = \begin{pmatrix} 14 & 8 & 3 \\ 8 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

and calculate its condition number using the maximum row-sum norm. 4