UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2009/10

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Dr. P. Howell

INSTRUCTIONS TO CANDIDATES: **Full marks for correct answers to 5 questions.**
A table of *Laplace* transforms is attached.

1. (a) Sketch the following functions and find their *Laplace* transforms:

(i) $f(t) = \begin{cases} 0, & \text{if } t < 1 \\ (t-1)^2 e^{-(t-1)}, & \text{if } t \geq 1 \end{cases}$ 2

(ii) $f(t) = \begin{cases} \sin 3\pi t, & \text{if } 0 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$ 3

(iii) $f(t) = \begin{cases} 0, & \text{if } t < 0 \\ t, & \text{if } 0 \leq t < 1 \\ 2-t, & \text{if } 1 \leq t < 2 \\ f(t-2), & \text{if } t \geq 2 \end{cases}$ 3

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$$(i) \quad \frac{2s-4}{s^2-s-6} \qquad\qquad (ii) \quad \frac{e^{-4s}}{s^2-3s} \qquad\qquad (iii) \quad \tan^{-1}\frac{1}{s}$$

2. (a) Use *Laplace* transforms to solve the initial value problem 10

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 3t + 1, \qquad y(0) = 0, \quad \frac{dy}{dt}(0) = 0$$

(b) Use *Laplace* transforms to solve the integral equation 6

$$y(t) = \cos t + 2\int_0^t \sin(t-u)y(u)du$$

3. (a) Find the *Fourier* series of the periodic function 5

$$f(x) = \begin{cases} -x, & \text{if } -\pi < x \leq 0 \\ x, & \text{if } 0 < x \leq \pi \end{cases} \qquad\qquad f(x+2\pi) = f(x)$$

(b) Hence evaluate the series 5

$$1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots$$

(c) Use the *Fourier* series of part (a) to find a particular solution of the differential equation 6

$$\frac{d^2y}{dx^2} + 2y = f(x)$$

where $f(x)$ is as in part (a).

4. (a) Let P_3 be the space of polynomials of degree at most three, i.e. $P_3 = \{p(x) : p(x) = a_0 + a_1x + a_2x^2 + a_3x^3\}$ where $a_0,\ a_1,\ a_2$ and a_3 are real numbers. Determine whether the set of polynomials

$$\{2 - 2x,\ 1 + x^2,\ -1 + 4x + 3x^2,\ 2x - x^3\}$$

forms a basis for P_3 8

(b) Find bases for the row space and column space, and hence the rank, of the matrix 8

$$\begin{pmatrix} 1 & -2 & 0 \\ -2 & -5 & 6 \\ 3 & -3 & 3 \\ 0 & 3 & -2 \end{pmatrix}$$

5. (a) Let P_1 be the space of polynomials of degree at most one. Use the *Gram-Schmidt* process to transform the standard basis $\{1,\ x\}$ for P_1 to an orthonormal one defined by the inner product

$$< p, q >= \sum_{i=1}^{5} p(x_i)q(x_i)$$

where $x_1 = -3,\ x_2 = -2,\ x_3 = -1,\ x_4 = 0,\ x_5 = 1.$ 8

(b) Let f be a function on $[-1, 3]$, taking the value $y_i = f(x_i)$ at $x = x_i,\ i = 1, 2, 3, 4, 5$ as given in the following table

x_i	y_i
-3	5
-2	3
-1	4
0	2
1	1

Find the least squares approximation to f in P_1 using the inner product defined in part (a). 8

6. Find the eigenvalues and corresponding eigenspaces of the matrix 8

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & 1 & 3 \end{pmatrix}$$

Show that A is diagonalisable over the reals and find the matrix P which diagonalises it. Hence or otherwise calculate A^4. 8

7. (a) Find the *Cholesky* decomposition ($A = LL^T$) of the matrix 5

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 13 & 7 \\ 1 & 7 & 6 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where 3

$$b = \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix}$$

(b) By considering the eigenvalues of the iteration matrix, or otherwise, show that the Jacobi method is guaranteed to converge for the system of linear equations 5

$$\begin{pmatrix} 1 & 1 & -2 \\ \frac{3}{2} & 1 & \frac{3}{2} \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

Use three (3) iterations of the method, with initial guess $x_1(0) = 0,\ x_2(0) = 0,\ x_3(0) = 0$, to obtain a numerical solution of the system. 3