UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2012/13

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Prof. T. Myers

INSTRUCTIONS TO CANDIDATES: **Full marks for correct answers to 5 questions.**
A table of *Laplace* transforms is attached.

1. (a) Sketch the graphs of the following functions and find their *Laplace* transforms:

(i) $f(t) = \begin{cases} 0, & \text{if } t < 5 \\ (t-5)e^{-2(t-5)}, & \text{if } t \geq 5 \end{cases}$ 2

(ii) $f(t) = \begin{cases} \sin 4\pi t, & \text{if } 0 \leq t < 2 \\ 0, & \text{otherwise} \end{cases}$ 3

(iii) $f(t) = \begin{cases} 0, & \text{if } t < 0 \\ t, & \text{if } 0 \leq t < 1 \\ 1, & \text{if } 1 \leq t < 2 \\ f(t-2), & \text{if } t \geq 2 \end{cases}$ 3

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$(i) \quad \dfrac{2s+2}{s^2+5s-6} \qquad (ii) \quad \dfrac{e^{-2s}}{s^2+4s+3} \qquad (iii) \quad \ln\left(\dfrac{1}{s(s+2)}\right)$

2. (a) Use *Laplace* transforms to solve the initial value problem 10

$$\frac{d^2y}{dt^2} - 4y = 12e^{-t}, \qquad y(0) = -1, \ \frac{dy}{dt}(0) = -2$$

(b) Use *Laplace* transforms to solve the integral equation 6

$$y(t) = t^2 - \int_0^t (t-u)y(u)du$$

3. (a) Sketch the graph of the following periodic function and find its *Fourier* series 6

$$f(x) = \begin{cases} 0, & \text{if } -\pi < x \leq -\frac{\pi}{2} \\ 1, & \text{if } -\frac{\pi}{2} < x \leq 0 \\ -1, & \text{if } 0 < x \leq \frac{\pi}{2} \\ 0, & \text{if } \frac{\pi}{2} < x \leq \pi \end{cases} \qquad f(x+2\pi) = f(x)$$

(b) Hence evaluate the series 4

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(c) The function $g : \mathbb{R} \to \mathbb{R}$ is an odd function of period 2π with *Fourier* series $\sum_{n=1}^{\infty} b_n \sin(nx)$. The function $h : \mathbb{R} \to \mathbb{R}$ is defined by

$$h(x) = g(x) + K$$

where K is a constant. Find the *Fourier* series of h. 6

4. (a) Determine whether the set of vectors

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \right\}$$

forms a basis for $\mathbb{R}^3$. 6

(b) Define span (S). What is the dimension of span (S) ? 4

(c) Is $(1,\ 2,\ 3)^T\ \in$ span (S) ?
For what value of a is $(1,\ a,\ 3)^T \in$ span (S) ? 6

5. (a) Let P_1 be the space of polynomials of degree at most one. Use the *Gram-Schmidt* process to transform the standard basis $\{1,\ x,\}$ for P_1 to an orthonormal one defined by the inner product

$$< p, q >= \int_{-1}^{0} p(x)q(x)\,dx$$

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(b) Find the least squares approximation to e^{-x} over the interval $[-1,\ 0]$ in P_1 using the inner product defined in part (a). 8

6. Find the eigenvalues and corresponding eigenspaces of the matrix 8

$$A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$$

Show that A is diagonalisable over the reals and find the matrix P which diagonalises it. Hence or otherwise solve the system of differential equations

$$\frac{d}{dt}X = AX, \qquad X(0) = (1,1)^T$$

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7. (a) Find the *Cholesky* decomposition ($A = LL^T$) of the matrix 5

$$A = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 5 & -2 \\ -2 & -2 & 9 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where 3

$$b = \begin{pmatrix} -1 \\ -11 \\ 16 \end{pmatrix}$$

(b) Define the condition number of a square matrix. Of what is it a measure ? 2
For the matrix A of part (a), use the *Cholesky* decomposition or otherwise to calculate its inverse, and then calculate its condition number using the maximum row-sum norm. 6