UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2013/14

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Prof. T. Myers

INSTRUCTIONS TO CANDIDATES: **Full marks for correct answers to 5 questions.**
A table of *Laplace* transforms is attached.

1. (a) Sketch the graphs of the following functions and find their *Laplace* transforms:

(i) $f(t) = \begin{cases} 0, & \text{if } t < 1 \\ e^{-3(t-1)}, & \text{if } t \geq 1 \end{cases}$ 2

(ii) $f(t) = \begin{cases} \sin 4\pi t, & \text{if } 0 \leq t < 2 \\ 0, & \text{otherwise} \end{cases}$ 3

(iii) $f(t) = \begin{cases} 2t, & \text{if } 0 \leq t < 1 \\ 2, & \text{if } 1 \leq t < 2 \\ 6-2t, & \text{if } 2 \leq t < 3 \\ 0, & \text{otherwise} \end{cases}$ 3

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$$(i)\ \frac{s+2}{s^2+2s+2} \qquad (ii)\ \frac{e^{-2s}}{s^2+5s+6} \qquad (iii)\ \tan^{-1}\left(\frac{1}{s}\right)$$

2. (a) Use *Laplace* transforms to solve the initial value problem 10

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 3 + 2t, \qquad y(0) = -1,\ \frac{dy}{dt}(0) = 4$$

(b) Use *Laplace* transforms to solve the integral equation 6

$$y(t) = t - \int_0^t e^{-(t-u)} y(u) du$$

3. (a) Sketch the graph of the following periodic function and find its *Fourier* series 6

$$f(x) = \begin{cases} -x, & \text{if } -\pi < x \leq 0 \\ x, & \text{if } 0 < x \leq \pi \end{cases} \qquad f(x+2\pi) = f(x)$$

(b) Hence evaluate the series 4

$$1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots$$

(c) The function $g : \mathbb{R} \to \mathbb{R}$ is an even function of period 2π with *Fourier* series $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$. The function $h : \mathbb{R} \to \mathbb{R}$ is defined by

$$h(x) = g(2x)$$

Find the *Fourier* series of h. 6

4. Let P_3 be the space of polynomials of degree at most three, i.e. $P_3 = \{p(x) : p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3\}$ where $a_0,\ a_1,\ a_2$ and a_3 are real numbers.

   (a) Determine whether the set of polynomials

   $$Q = \{2 - x,\ 3 + x^2,\ -2 + 9x + 6x^2 - 2x^3,\ x + 2x^3\}$$

   forms a basis for P_3 8

   (b) Define span (Q). What is the dimension of span (Q) ? 3

   (c) Is $3 - 4x + 3x^2 - x^3 \in$ span (Q) ?
   For what value(s) of a is $1 + ax^2 + x^3 \in$ span (Q) ? 5

5. (a) Let P_1 be the space of polynomials of degree at most one. Use the *Gram-Schmidt* process to transform the standard basis $\{1,\ x\}$ for P_1 to the orthonormal one defined by the inner product

   $$< p, q >= \sum_{i=1}^{5} p(x_i) q(x_i)$$

   where $x_1 = 0,\ x_2 = 1,\ x_3 = 2,\ x_4 = 3,\ x_5 = 4.$ 8

   (b) Let f be a function on $[0, 4]$, taking the value $f(x_i)$ at $x = x_i,\ i = 1, 2, 3, 4, 5$ as given in the following table

x_i	$f(x_i)$
0	-2
1	0
2	3
3	6
4	8

   Find the least squares approximation to f in P_1 using the inner product defined in part (a). 8

6. Find the eigenvalues and corresponding eigenspaces of the matrix 8

   $$A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$$

   Show that A is diagonalisable over the reals and find the matrix P which diagonalises it. Hence or otherwise solve the system of differential equations

   $$\frac{d}{dt} X = AX, \qquad X(0) = (1, -1)^T$$

   8

7. (a) Find the *Cholesky* decomposition ($A = LL^T$) of the matrix 5

$$A = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 2 & 1 \\ 0 & 1 & 20 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where 3

$$b = \begin{pmatrix} -2 \\ 0 \\ -8 \end{pmatrix}$$

(b) Define the condition number of a square matrix. Of what is it a measure ? 2

Determine the inverse of the matrix A of part (a), and then calculate its condition number using the maximum row-sum norm. 6