UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2006/07

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 80 %

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: **Full marks for correct answers to 5 questions.**

1. (a) Sketch the following functions and find their *Laplace* transforms:

(i) $f(t) = \begin{cases} 0, & \text{if } t < 4 \\ (t-4)^2, & \text{if } t \geq 4 \end{cases}$ 2

(ii) $f(t) = \begin{cases} \sin 3\pi t, & \text{if } 0 \leq t < 2 \\ 0, & \text{otherwise} \end{cases}$ 3

(iii) $f(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } 0 \leq t < 1 \\ 2, & \text{if } 1 \leq t < 2 \\ f(t-2), & \text{if } t \geq 2 \end{cases}$ 3

(b) Find the inverse *Laplace* transforms of the following 2,3,3

$(i)\ \dfrac{s+1}{s^2-4s+3} \qquad (ii)\ \dfrac{e^{-s}}{s^2+2s+1} \qquad (iii)\ \ln \dfrac{s^2}{s^2+1}$

2. (a) Use *Laplace* transforms to solve the initial value problem 10

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = f(t), \qquad y(0) = 0, \quad \frac{dy}{dt}(0) = 0$$

where

$$f(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq 1 \\ e^{-3(t-1)}, & \text{otherwise} \end{cases}$$

(b) Use *Laplace* transforms to solve the integral equation 6

$$y(t) = 1 + t + \int_0^t (t-u)y(u)du$$

3. (a) Find the *Fourier* series of the periodic function 6

$$f(x) = \begin{cases} 1+x, & \text{if } -1 < x \leq 0 \\ 1-x, & \text{if } 0 < x \leq 1 \end{cases} \qquad f(x+2) = f(x)$$

(b) Hence evaluate the series 4

$$1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots$$

(c) Use the *Fourier* series of part (a) to find a particular solution of the differential equation 6

$$\frac{dy}{dx} + 2y = f(x)$$

where $f(x)$ is as in part (a).

4. Let P_2 be the space of polynomials of degree at most two, i.e. $P_2 = \{p(x) : p(x) = a_0 + a_1 x + a_2 x^2\}$ where a_0, a_1 and a_2 are real numbers.

   (a) Determine whether the set of polynomials

   $$Q = \{1 + 2x,\ 2x + x^2,\ 4 + 2x - 3x^2\}$$

   forms a basis for P_2 6

   (b) What is dim span (Q) ? 4

   (c) Is $-3 + 3x^2 \ \in$ span (Q) ? 6

5. (a) Let P_1 be the space of polynomials of degree at most one. Use the *Gram-Schmidt* process to transform the standard basis $\{1,\ x\}$ for P_1 to an orthonormal one defined by the inner product

   $$< p, q >= \sum_{i=1}^{5} p(x_i)q(x_i)$$

   where $x_1 = -2,\ x_2 = -1,\ x_3 = 0,\ x_4 = 1,\ x_5 = 2.$ 8

   (b) Let f be a function on $[-2, 2]$, taking the value $f(x_i)$ at $x = x_i,\ i = 1, 2, 3, 4, 5$ as given in the following table

x_i	$f(x_i)$
-2	-2
-1	1
0	2
1	6
2	8

   Find the least squares approximation to f in P_1 using the inner product defined in part (a). 8

6. Find the eigenvalues and corresponding eigenspaces of the matrix 8

   $$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix}$$

   Is A diagonalisable over the reals ? If so, find the matrix P which diagonalises it. Calculate A^4. 8

7. (a) Find the *Cholesky* decomposition ($A = LL^T$) of the matrix 5

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & -6 \\ 3 & -6 & 10 \end{pmatrix}$$

and hence solve the system of equations $Ax = b$ where 5

$$b = \begin{pmatrix} -3 \\ 7 \\ -10 \end{pmatrix}$$

(b) Define the condition number of a square matrix. Of what is it a measure ? 2

For the matrix A of part (a) verify that its inverse is given by

$$A^{-1} = \begin{pmatrix} 14 & 2 & -3 \\ 2 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}$$

and calculate its condition number using the maximum row-sum norm. 4