UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering
Department of Mathematics & Statistics

MID TERM ASSESSMENT PAPER

MODULE CODE: MA4003

SEMESTER: Autumn 2012/13

MODULE TITLE: Engineering Mathematics 3

DURATION OF EXAMINATION: 45 minutes

LECTURER: Dr. M. Burke

PERCENTAGE OF TOTAL MARKS: 20 %

Colour: Yellow

INSTRUCTIONS TO CANDIDATES: **Answer all questions. All questions carry equal marks. Use the Answer Sheet below.**

ANSWER SHEET

STUDENT'S NAME:
STUDENT'S ID NUMBER:

For each question, place an "**X**" in the box of your choice.

Question	a	b	c	d	e	Do not write in this column
1				**X**		
2					**X**	
3	**X**					
4			**X**			
5				**X**		
6				**X**		
7					**X**	
8					**X**	
9		**X**				
10			**X**			

Table of Laplace Transforms

$f(t),\ t \geq 0$	$F(s) = \mathcal{L}[f(t)]$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\frac{1}{a-b}(e^{at} - e^{bt})$	$\frac{1}{(s-a)(s-b)}$
$\frac{a}{a-b}e^{at} - \frac{b}{a-b}e^{bt}$	$\frac{s}{(s-a)(s-b)}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\int_0^t f(\tau)\, d\tau$	$\frac{1}{s}F(s)$
$e^{at}f(t)$	$F(s-a)$
Heaviside $u_a(t)$	$\frac{e^{-as}}{s}$
$f(t-a)u_a(t)$	$e^{-as}F(s)$
Ramp $R(t-a)$	$\frac{e^{-as}}{s^2}$
$tf(t)$	$-F'(s)$
$\dfrac{f(t)}{t}$	$\displaystyle\int_s^\infty F(\sigma)\, d\sigma$
$(f * g)(t) \equiv \displaystyle\int_0^t f(t-\tau)g(\tau)\, d\tau$	$F(s)G(s)$
$f(t) = f(t+p)$	$\dfrac{1}{1-e^{-sp}} \displaystyle\int_0^p f(t)e^{-st}\, dt$

All $f(t)$ are defined for $t \geq 0$.

1. The *Laplace* Transform of $(t-1)^3$ is

   $(a)\ \dfrac{6}{s^4}e^{-s}\quad (b)\ \dfrac{2}{s-1}e^{-3s}\quad (c)\ \dfrac{1}{s^4}-\dfrac{3}{s^3}+\dfrac{3}{s^2}-\dfrac{1}{s}\quad (d)\ \dfrac{6}{s^4}-\dfrac{6}{s^3}+\dfrac{3}{s^2}-\dfrac{1}{s}\quad (e)\ \dfrac{6}{(s-1)^4}$

2. The *Laplace* Transform of $e^{-3t}(4t-1)$ is

   $(a)\ \dfrac{4s-1}{(s+3)^2}\quad (b)\ \dfrac{4-s}{s^2(s+3)}\quad (c)\ \dfrac{4}{s^2-9}\quad (d)\ \dfrac{4-s}{(s+3)^2}\quad (e)\ \dfrac{1-s}{(s+3)^2}$

3. The *Laplace* Transform of $f(t) = \sin(2t-2)u_1(t)$ is

   $(a)\ \dfrac{2}{s^2+4}e^{-s}\quad (b)\ \dfrac{2e^{-1}}{s(s^2+4)}\quad (c)\ \dfrac{1}{s^2+1}e^{-s}\quad (d)\ \dfrac{s}{s^2+4}e^{-s}\quad (e)\ \dfrac{2}{s^2+4}e^{-2s}$

4. The inverse *Laplace* transform of $\frac{1}{s+3}e^{-2s}$ is

   $(a)\ 3e^{-2t}\quad (b) 3e^{-2t}\quad (c)\ e^{-3(t-2)}u_2(t)\quad (d)\ (t-2)u_2(t)\quad (e)\ e^{-3t}u_2(t)$

5. The inverse *Laplace* transform of $\frac{s+1}{s^2+2s+5}$ is

   $(a)\ e^{-5t}\quad (b)\ \dfrac{1}{4}(e^{-t}-e^{-5t})\quad (c)\ e^{t}\cos 2t\quad (d)\ e^{-t}\cos 2t\quad (e)\ e^{-t}\sin 2t$

6. The convolution of e^t with e^{-t}; (also denoted by $e^t * e^{-t}$) is given by

   $(a)\ 1\quad (b)\ t\quad (c)\ \sin t\quad (d)\ \sinh t\quad (e)\ \cosh t$

7. The function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(x+2\pi) = f(x)$. The period of $f(x/2)$ is

   $(a)\ \pi\quad (b)\ 2\quad (c)\ 2\pi\quad (d)\ 4\quad (e)\ 4\pi$

8. The functions $f(x) = x^2 - x^5$ and $g(x) = x\sin x$ defined on $-1 < x < 1$ have the property that
   (a) both are even (b) both are odd (c) f is odd and g is even
   (d) f is even and g is odd (e) at least one is neither even nor odd

9. The function $f(x) = \begin{cases} 1, & \text{if } -\pi < x < 0 \\ 0, & \text{if } 0 < x < \pi \end{cases}$ is periodic with period 2. It has a *Fourier* Series $\frac{1}{2} + \sum_{n=1}^{\infty} b_n \sin(nx)$; b_3 is given by

   $(a)\ -\dfrac{4}{3\pi}\quad (b)\ -\dfrac{2}{3\pi}\quad (c)\ 0\quad (d)\ \dfrac{2}{3\pi}\quad (e)\ \dfrac{4}{3\pi}$

10. The coefficient a_0 in the *Fourier* Series for the periodic function $f(x) = x\cosh x$ if $-1 < x < 1$ with period 2 has the value

    $(a)\ 2(\sinh 1 - \cosh 1)\quad (b)\ -\sinh 1\quad (c)\ 0\quad (d)\ \cosh 1\quad (e)\ 2(\sinh 1 - \cosh 1 + 1)$