1. From the tables, we have

$$(a)\ \frac{2!}{s^3}, \qquad (b)\ \frac{s}{s^2+6^2}, \qquad (c)\ \frac{1}{s-3}, \qquad (d)\ \frac{1}{(s+1)^2}\ \text{(using } e^{at}f(t) \leftrightarrow F(s-a)\text{)}$$

$$(e)\ \frac{2}{s}+\frac{10s}{(s^2+25)^2}\ \text{(using } tf(t) \leftrightarrow -F'(s) = -\frac{dF(s)}{ds}\text{)}, \qquad (f)\ \frac{1}{s}e^{-4s}, \qquad (g)\ \text{same as (f)}$$

2. (see graphs).
(a)

$$f(t) = \cos 2(t-2\pi)\, u_{2\pi}(t) \leftrightarrow e^{-2\pi s}\frac{s}{s^2+2^2}$$

(b)

$$f(t) = \sin t\, u_0(t) - \sin(t-2\pi)\, u_{2\pi}(t) \leftrightarrow \frac{1}{s^2+1} - \frac{1}{s^2+1}e^{-2\pi s}$$

(c) Square wave: Note that

$$f(t) = \begin{cases} 1, & \text{if } 0 \le t < 1 \\ -1, & \text{if } 1 \le t < 2 \\ 0, & \text{elsewhere} \end{cases} = u_0(t) - 2u_1(t) + u_2(t)$$

Thus we have

$$F(s) = \frac{1}{s} - 2\frac{1}{s}e^{-s} + \frac{1}{s}e^{-2s} = \frac{1}{s}[1-2e^{-s}+e^{-2s}] = \frac{1}{s}\left(1-e^{-s}\right)^2$$

(d) Saw-tooth or triangular wave: Note that

$$f(t) = \begin{cases} t, & \text{if } 0 \le t < 1 \\ 2-t, & \text{if } 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases} = R(t) - 2R(t-1) + R(t-2)$$

where $R(t)$ is the ramp function (see Tables). We have

$$F(s) = \frac{1}{s^2} - 2\frac{1}{s^2}e^{-s} + \frac{1}{s^2}e^{-2s} = \frac{1}{s^2}[1-2e^{-s}+e^{-2s}] = \frac{1}{s^2}\left(1-e^{-s}\right)^2$$

3. All $f(t), t \ge 0$: From tables we get

$$(a)\ e^{-t}, \qquad (b)\ \frac{1}{3}\sin 3t, \qquad (c)\ \cosh 3t$$

$$(d)\ \text{By partial fraction expansion}\ \frac{3/2}{s+3} - \frac{1/2}{s+1} \leftrightarrow \frac{3}{2}e^{-3t} - \frac{1}{2}e^{-t}$$

$$(e)\ \frac{4}{(s+1)^2} \leftrightarrow 4te^{-t}, \qquad (f)\ u_4(t)$$

4. (See graphs).

(a) Rectified sine wave: define

Then

$$f(t) = g(t) + g(t-\tfrac{\pi}{3}) + g(t-2\tfrac{\pi}{3}) + g(t-3\tfrac{\pi}{3}) + \cdots$$

We have

$$G(s) = \frac{3}{s^2+3^2} + \frac{3}{s^2+3^2}e^{-\frac{\pi}{3}s} = \frac{3}{s^2+9}(1+e^{-\frac{\pi}{3}s})$$

and therefore

$$\begin{aligned}
F(s) &= G(s) + G(s)e^{-\frac{\pi}{3}s} + G(s)e^{-2\frac{\pi}{3}s} + G(s)e^{-3\frac{\pi}{3}s} + \cdots \\
&= G(s)[1 + e^{-\frac{\pi}{3}s} + e^{-2\frac{\pi}{3}s} + e^{-3\frac{\pi}{3}s} + \cdots] \\
&= G(s)\frac{1}{1-e^{-\frac{\pi}{3}s}} \\
&= \frac{3}{s^2+9}\left(\frac{1+e^{-\frac{\pi}{3}s}}{1-e^{-\frac{\pi}{3}s}}\right)
\end{aligned}$$

(b) Let $g(t)$ stand for the square wave of Q2(c). Then

$$f(t) = g(t) + g(t-2) + g(t-4) + g(t-6) + \cdots$$

and therefore

$$\begin{aligned}
F(s) &= G(s) + G(s)e^{-2s} + G(s)e^{-4s} + G(s)e^{-6s} + \cdots \\
&= G(s)[1 + e^{-2s} + e^{-4s} + e^{-6s} + \cdots] \\
&= G(s)\frac{1}{1-e^{-2s}} \\
&= \frac{1}{s}\left(\frac{1-2e^{-s}+e^{-2s}}{1-e^{-2s}}\right) \\
&= \frac{1}{s}\left(\frac{1-e^{-s}}{1+e^{-s}}\right)
\end{aligned}$$

(c) Let $g(t)$ stand for the saw-tooth wave of Q2(d). Then

$$f(t) = g(t) + g(t-2) + g(t-4) + g(t-6) + \cdots$$

and therefore

$$\begin{aligned}
F(s) &= G(s) + G(s)e^{-2s} + G(s)e^{-4s} + G(s)e^{-6s} + \cdots \\
&= G(s)[1 + e^{-2s} + e^{-4s} + e^{-6s} + \cdots] \\
&= G(s)\frac{1}{1-e^{-2s}} \\
&= \frac{1}{s^2}\left(\frac{1-2e^{-s}+e^{-2s}}{1-e^{-2s}}\right) \\
&= \frac{1}{s^2}\left(\frac{1-e^{-s}}{1+e^{-s}}\right)
\end{aligned}$$

5. All $f(t), t \geq 0$: We get
(a)

By completion of the square $\dfrac{1}{(s+2)^2+2^2} \leftrightarrow \dfrac{1}{2}e^{-2t}\sin 2t$ (using $e^{at}f(t) \leftrightarrow F(s-a)$)

(b)

$$\cos 2(t-1)u_1(t)$$

(c)

By partial fraction expansion $\dfrac{1}{s+1} - \dfrac{5}{s+2} + \dfrac{5}{s+3} \leftrightarrow e^{-t} - 5e^{-2t} + 5e^{-3t}$

(d)

$$\begin{aligned} \frac{2}{s}\left(\frac{1}{1+e^{-2s}}\right) &= \frac{2}{s}(1-e^{-2s}+e^{-4s}-e^{-6s}+\cdots) \\ &\leftrightarrow 2u_0(t) - 2u_2(t) + 2u_4(t) - 2u_6(t) + \cdots \end{aligned}$$