

1. Identify the period of the following periodic functions and find their *Fourier* Series representations.

(a)

$$f(x) = x \text{ if } -2 < x \leq 2, \quad f(x+4) = f(x)$$

(b)

$$f(x) = |\sin x|$$

(c)

$$f(x) = \begin{cases} 0, & \text{if } -1 < x < 0 \\ x^2, & \text{if } 0 \leq x \leq 1 \end{cases} \quad f(x+2) = f(x)$$

2. Find (i) the even extension and (ii) the odd extension of the following function and hence determine its (*Fourier*) Cosine Series representation and Sine Series representation respectively.

$$f(x) = x^2, \text{ if } 0 \leq x \leq 1$$

3. Evaluate the following series

(a)

$$\frac{1}{1 \times 3} + \frac{1}{5 \times 7} + \frac{1}{9 \times 11} + \dots$$

using FS of Q1(a)

(b)

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots$$

using Cosine Series of Q2

(c)

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

using Cosine Series of Q2.

4. Using a *Fourier* Series representation of the right hand sides, find a particular solution of each of the following differential equations

(a)

$$\frac{dy}{dx} + 2y = \begin{cases} -1, & \text{if } -1 \leq x < 0 \\ 1, & \text{if } 0 \leq x < 1 \end{cases}$$

(b)

$$\frac{d^2y}{dx^2} + y = \begin{cases} x, & \text{if } 0 \leq x < 1 \\ 2-x, & \text{if } 1 \leq x < 2 \end{cases}$$