1. Let $V = \mathbf{R}^{2\times 2}$, the space of 2×2 matrices with real entries and, as usual, let the i, j-th entry in vector (matrix) A be given by (a_{ij}). Prove that $< A, B >= a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$ defines an inner product. Let $||A||$ denote the induced norm.
For
$$A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & 3 \\ 2 & 4 \end{pmatrix}$$
   (a) Verify the *Cauchy-Schwartz* inequality.
   (b) Find $||A||$.
   (c) Find the angle between A and B.
   (d) What is the dimension of V ? Give an orthonormal basis for V.

2. Let $V = \mathbf{R}^{n\times n}$. For $A \in V$, show that
$$||A|| = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}|$$
defines a norm, the "maximum row sum" norm. For example
$$\begin{aligned} A &= \begin{pmatrix} 2 & -3 \\ 5 & 2 \end{pmatrix} \\ \Rightarrow ||A|| &= \max\{|2| + |-3|,\ |5| + |2|\} = 7 \end{aligned}$$

3. Use the *Gram-Schmidt* orthonormalisation procedure to generate orthonormal bases in the following cases:
   (a) $V = \mathbf{R}^3$ with the standard inner product. Start with the basis $\{(1, 1, 0)^T,\ (1, -1, 0)^T,\ (1, 2, 1)^T\}$
   (b) $V = \mathbf{R}^3$ with inner product $< u, v >= u_1v_1 + 2u_2v_2 + 3u_3v_3$. Start with basis $\{(1, 1, 0)^T,\ (1, -1, 0)^T,\ (0, 0, 1)^T\}$
   (c) $V = P_2$ with inner product $< p, q >= \sum_{i=1}^{5} p(x_i)q(x_i)$ where $x_i = i - 3$. Start with the standard basis for P_2.
   (d) $V = P_3$ with inner product $< p, q >= \int_{-2}^{2} p(x)q(x)dx$. Start with the standard basis for P_3.

4. Find the least squares approximation of e^x over the interval $[0, 1]$ by a quadratic function using the norm induced by $< f, g >= \int_0^1 f(x)g(x)dx$.

5. Let f be a function on $[-2, 2]$, taking the value $f(x_i)$ at $x = x_i,\ i = 1, 2, 3, 4, 5$ as given in the following table

x_i	-2	-1	0	1	2
$f(x_i)$	0	-1	0	2	4

Find the least squares approximation to f in P_1 and P_2 using the norm induced by the inner product of Q3 part (c).