1. Find the eigenvalues and corresponding eigenspaces of the matrices

$$A_1 = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & -3 & -1 \\ 1 & 0 & -1 \\ -3 & -3 & 2 \end{pmatrix}, \qquad A_4 = \begin{pmatrix} 0 & 2 & -2 \\ 2 & -3 & 4 \\ -2 & 4 & -5 \end{pmatrix}$$

2. Which of the matrices of Question (1) are diagonalisable ? Of those that are, find a diagonalising matrix P.

3. Which of the matrices of Question (1) are orthogonally diagonalisable ? Of those that are, show that $P^{-1} = P^T$.

4. Using the matrices of Question (1), compute

$$A_1^5, \qquad A_4^4$$

5. Consider the system of linear differential equations

$$\begin{aligned} \frac{dx}{dt} &= 2y - 2z \\ \frac{dy}{dt} &= 2x - 3y + 4z \\ \frac{dz}{dt} &= -2x + 4y - 5z \end{aligned}$$

with initial conditions

$$x(0) = 2, \qquad y(0) = 0, \qquad z(0) = -1$$

(a) Write the system in vector matrix notation.

(b) By diagonalising the system matrix, and thereby using an appropriate linear transformation (change of variable), convert the system to 3 "decoupled " first order differential equations.

(c) Solve this new system, and hence obtain the solution to the original system.