MA4006: Exercise Sheet 1

Please attempt these questions before the tutorials in Week 2.

1. Sketch the curves represented by the following parametric equations:

(i) $\mathbf{r}(t) = (2\cos\pi t, \sin\pi t, 0)$, (ii) $\mathbf{r}(t) = (\sin\pi t, 0, 0)$, (iii) $\mathbf{r}(t) = (t, |t|, 0)$

(iv) $\mathbf{r}(t) = (t, -t, 0), \quad -\infty < t \leq 0$, (v) $\mathbf{r}(t) = (t, -t^2, 0), \quad 0 < t \leq \infty$.

2. Determine the derivatives $\dfrac{d\mathbf{r}}{dt}$ and $\dfrac{d^2\mathbf{r}}{dt^2}$ of the following vectors:

(i) $\mathbf{r}(t) = (2\cos\pi t, \sin\pi t, 0)$, (ii) $\mathbf{r}(t) = (t, t, e^t)$, (iii) $\mathbf{r}(t) = (t^2, t^3 - t, 0)$.

3. In the following, (a) compute $\mathbf{f} \cdot \mathbf{g}$ and differentiate the resulting real-valued function and (b) compute $(\mathbf{f} \cdot \mathbf{g})'$ using the differential rule (iii) in Section 1.4 of the lecture notes.

(i) $\mathbf{f}(t) = e^t\mathbf{j} + 2\mathbf{k}, \quad \mathbf{g}(t) = \cos t\mathbf{i} + 2\mathbf{j} + t^2\mathbf{k}$

(ii) $\mathbf{f}(t) = -4\cos t\mathbf{k}, \quad \mathbf{g}(t) = -t^2\mathbf{i} + 4\sin t\mathbf{k}$.

4. In the following, (a) compute $\mathbf{f} \times \mathbf{g}$ and differentiate the resulting real-valued function and (b) compute $(\mathbf{f} \times \mathbf{g})'$ using the differential rule (iv) in Section 1.4 of the lecture notes.

(i) $\mathbf{f}(t) = t\mathbf{i} + \mathbf{j} + 4\mathbf{k}, \quad \mathbf{g}(t) = \mathbf{i} - \cos t\mathbf{j} + t\mathbf{k}$

(ii) $\mathbf{f}(t) = -9\mathbf{i} + t^2\mathbf{j} + t^2\mathbf{k}, \quad \mathbf{g}(t) = e^t\mathbf{i}$.

5. Find the unit tangent vector to the curve $\mathbf{r} = (3, t, t^2)$.

6. Consider the vector valued function

$$\mathbf{r}(t) = \begin{cases} (t, -t^3, 0), & \text{if } -1 \leq t \leq 1 \\ (t, 2 - 3t, 0), & \text{if } 1 < t \leq 2. \end{cases}$$

Find its derivative, and hence find the unit tangent vector to the curve defined by $\mathbf{r}(t)$ and comment on its smoothness.

7. Consider the vector valued function

$$\mathbf{r}(t) = \begin{cases} (t^2, t, 0), & \text{if } -1 \leq t \leq 1 \\ (1, t, 0), & \text{if } 1 < t \leq 2. \end{cases}$$

Find its derivative, and hence find the unit tangent vector to the curve defined by $\mathbf{r}(t)$ and comment on its smoothness.

8. Find the intrinsic equation of the curve $\mathbf{r} = (a\cos t, a\sin t, bt)$, $(0 \leq t \leq 3\pi)$.

9. Find an expression for the arclength of the curve expressed perimetrically as $\mathbf{r}(t) = (0, 5\cos t, 5\sin t)$ where $0 \leq t \leq 2\pi$. Using the arclength find the intrinsic equation of the curve represented by $\mathbf{r}(t)$. Find an expression for the curvature of this curve at any point.

10. Sketch the curve $\mathbf{r}(t) = (t, t+1)$ $0 \leq t \leq 1$ in two dimensions and find an expression for its arclength. Hence find the arclength from $t = 0$ to $t = 1$. Write the equation for the curve in intrinsic form.

11. Sketch the curve $\mathbf{r}(t) = 4t\mathbf{i} + t^2\mathbf{j}$ in two dimensions and find the velocity, speed and acceleration at time t. Hence sketch the velocity and acceleration vectors when $t = -1$.