Maths Enrichment: Yet More Sequences

1. Find all positive integers $n \geq 2$ such that the inequality

$$x_1x_2 + x_2x_3 + x_3x_4 + \ldots + x_{n-1}x_n \leq \frac{n-1}{n}\left(x_1^2 + x_2^2 + \ldots + x_n^2\right)$$

   is satisfied for all positive real numbers $x_1, x_2, \ldots, x_n$.

2. Consider the sequences $x_0, x_1, \ldots x_{2013}$ of integers satisfying

$$\begin{aligned} x_0 &= 0 \\ |x_n| &= |x_{n-1} + 1| \qquad n = 1, 2, \ldots, 2013 \end{aligned}$$

   Find the minimum value of the expression $|x_1 + x_2 + \ldots + x_{2013}|$.

3. The positive integers $x_1, x_2, \ldots, x_7$ satisfy the conditions

$$\begin{aligned} x_6 &= 144 \\ x_{n+3} &= x_{n+2}(x_{n+1} + x_n) \qquad n = 1, 2, 3, 4 \end{aligned}$$

   Find x_7.

4. Define a sequence by

$$\begin{aligned} x_0, x_1 &\in \mathbb{R} \\ x_{n+2} &= \frac{1 + x_{n+1}}{x_n} \end{aligned}$$

   Find x_{2013}.

5. Consider the sequence defined by

$$\begin{aligned} a_0, a_1 &= 1 \\ a_{n+2} &= 14a_{n+1} - a_n \end{aligned}$$

   Prove that $2a_n - 1$ is a perfect square.