1. (a) $x_e = 0$ or $\ln a$ with multipliers $f'(x_e) = a$ and $1 - \ln a$ respectively. The former is stable if $(0 <)a < 1$ while the latter is stable if $1 < a < e^2 \approx 7.3891$.

   (b) At $a = e^2$, $f'(x_e) = -1$ and thus a period doubling bifurcation from a fixed point to a 2-cycle occurs.
   The points of the 2-cycle x_1, x_2 are obtained by solving

$$\frac{f(f(x)) - x}{f(x) - x} = \frac{a^2 x e^{-x} e^{-axe^{-x}} - x}{axe^{-x} - x} = 0$$

   while at the period doubling bifurcation to a 4-cycle, the multiplier of the 2-cycle satisfies

$$\frac{d\,(f(f(x)))}{dx} = a(1 - x_1)e^{-x_1} a(1 - x_2)e^{-x_2} = -1$$

   solving these conditions yields (numerically)

$$a = 12.5092, \qquad x_1 = .7016, x_2 = 4.3513$$

   (c) $a3 \approx 22.25217854$.

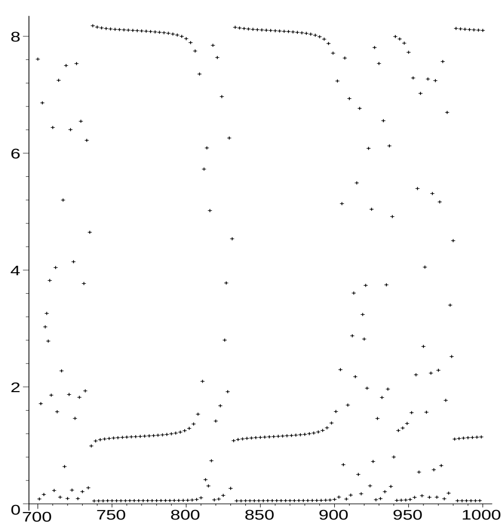

Figure 1: Intermittency for Ricker Map with $a = 22.235$

2. (a) Since $|f'(x)| = \mu$ for all x, we get $LE(x) = \lim_{n\to\infty} 1/n(n \ln |\mu|| = \ln |\mu|$ which is positive for both given values of μ.

   (b) I got $LE(0.14285) = 0.4949018737$ using 10,000 iterations.

   (c) I got $LE(0.30969) = 0.4630437779$ using 10,000 iterations.

3. For this *Ricker* map, $a_0 = 30$ and $x_e = \ln 30 \approx 3.4012$. The linearised map is described by

$$A = \frac{\partial f}{\partial x}(x_e, a_0) = a_0(1 - x_e)e^{-x_e} \approx -2.4012, \qquad B = \frac{\partial f}{\partial a}(x_e, a_0) = x_e e^{-x_e} \approx 0.1138$$

Hence for superstable control

$$K = -\frac{A}{B} = a_0(1 - 1/x_e) \approx 21.1796$$

The *OGY* control law is therefore

$$a = \begin{cases} a_0 + K(x - x_e), & \text{if } |x - x_e| < 0.05 \\ a_0, & \text{otherwise} \end{cases} \quad (1)$$

$$= \begin{cases} 30 + 21.1796(x - 3.4012), & \text{if } |x - 3.4012| < 0.05 \\ 30, & \text{otherwise} \end{cases} \quad (2)$$

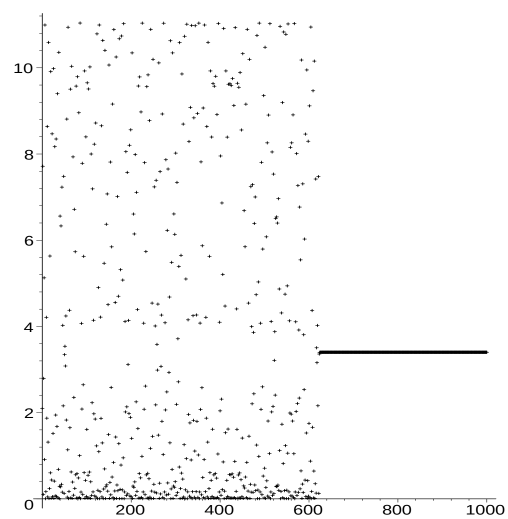

Figure 2: Orbit of Ricker Map with $a_0 = 30$. No control is used until the OGY strategy is switched on at time t=501

If $\Delta a = 2\% = 0.6$, then the radius of the active region satisfies

$$\epsilon \leq \frac{\Delta a}{K} = \frac{0.6}{21.1796} \approx 0.02833$$

and the *OGY* control law is

$$a = \begin{cases} 30 + 21.1796(x - 3.4012), & \text{if } |x - 3.4012| < 0.02833 \\ 30, & \text{otherwise} \end{cases}$$