MS4018 Problem Sheet 3

1. Find the fixed points of the following 2-d flows and classify them:

   (a) $\dot{x} = 2x - 3y, \qquad \dot{y} = 3x - 3y$

   (b) $\dot{x} = x - 2y, \qquad \dot{y} = -4x + 3y$

   (c) $\dot{x} = 2x(1 - x - 3y), \qquad \dot{y} = 7y(1 - y - 1.5x)$

2. Find a classification scheme for the fixed points of linear 2-d **maps** in terms of (i) their eigenvalues, (ii) the coefficients of their characteristic polynomials. If the characteristic polynomial is denoted by $\lambda^2 - a\lambda + b$, show the $a - b$ plane may be partitioned in terms of classification types.

3. Find the fixed points of the following 2-d maps and classify them:

   (a) $x' = 0.5x + 0.4y, \qquad y' = -0.25x + y$

   (b) $x' = 2x - 1.125y, \qquad y' = x - 0.25y$

   (c) $x' = -x^2 + 0.4y, \qquad y' = x$

4. Can periodic orbits of 2-d maps be classified as per Q2 ? If so, how ?

5. The Hénon map is defined by

$$x' = p - x^2 + qy, \qquad y' = x$$

   (For instance the map in Q3(c) is a Hénon map with $p = 0$ and $q = 0.4$).

   (a) With $p = 0.43$, $q = 0.4$, show that $(0.7, -0.1)$ and $(-0.1, 0.7)$ form a 2-cycle. Classify this cycle.

   (b) Set $q = 0.4$; show that for $0.27 < p < 0.85$, the map has a 2-cycle which is a sink.