1. For each of the following 1-d flows, calculate the fixed point(s) as a function of the parameter a, show that a bifurcation occurs at a certain critical value of $a = a_c$, determine the type of bifurcation, and sketch the bifurcation diagram x_e vs a.

   (a) $\dot{x} = 1 - ax + x^2$

   (b) $\dot{x} = a + x - e^{-x}$

   (c) $\dot{x} = x - ax(1 - x)$

   (d) $\dot{x} = x + \frac{ax}{1+x^2}$

   (e) $\dot{x} = ax - \frac{x}{1+x^2}$

2. (*Scaling* as a tool: scaling is very similar to nondimensionaling. We are primarily interested in it as a means of reducing the number of parameters).
   The equation
   $$\dot{N} = rN(1 - \frac{N}{K}) - H$$
   where r, K and $H \geq 0$ is a model of a fishery with harvesting. $N(t)$ is the population size. In the absence of harvesting (H=0), the population grows logistically (r is the *Malthusian* growth rate, K is the *carrying capacity.*) H is a constant rate of harvesting, irrespective of the population size.

   (a) Show that the system can be rescaled as
   $$\frac{dx}{d\tau} = x(1 - x) - h$$
   for suitably defined dimensionless quantities x, τ and h.

   (b) Find the fixed points as a function of the parameter h, and show that a bifurcation occurs; classify it and determine the critical value $h = h_c$.

   (c) What's happening when $h < h_c$? $h > h_c$?
   Is there anything wrong with this model ?

3. For the system
   $$\dot{x} = y - 2x, \qquad \dot{y} = a + x^2 - y$$

   (a) Sketch the nullclines

   (b) Find and classify the zero-eigenvalue bifurcations that occur as a varies.

4. Find and classify all zero-eigenvalue bifurcations for the 2-d flow
   $$\dot{x} = y - ax, \qquad \dot{y} = -by + \frac{x}{1 + x}$$

5. What type of bifurcation occurs at the origin when $a_c = 0$ for
   $$\dot{x} = y + ax, \qquad \dot{y} = -x + ay - x^2y$$

6. (a) Show that the predator-prey model

$$\dot{x} = x\left(b - x - \frac{y}{1+x}\right), \qquad \dot{y} = y\left(\frac{x}{1+x} - ay\right)$$

($x,\ y \geq 0$ and $a,\ b > 0$) has a positive fixed point $\mathbf{x}_e$ for all allowable values of a and b.

(b) Show that a *Hopf* bifurcation occurs at $\mathbf{x}_e$ for

$$a = a_c = \frac{4(b-2)}{b^2(b+2)}$$

when $b > 2$.

(c) Is the bifurcation super- or subcritical ?