MS4018 Problem Sheet 7

1. The 1-d map *Ricker* map
$$x' = axe^{-x}$$
was introduced originally to model the salmon population in certain Canadian rivers. x is a measure of the size of the population on a yearly basis, while a is a growth parameter. The map is an example of a unimodal map, and thus has the features associated with that class of map: e.g. a period doubling cascade, chaotic behaviour for certain parameter values, intermittency (see (c) below), etc.

   (a) Find the fixed points of the map and investigate their stability as a function of the parameter a.

   (b) For what value of a does the relevant fixed point bifurcate to a 2-cycle ? the 2-cycle to a 4-cycle ?

   (c) Using Maple, estimate the value of $a = a_3$ at which a 3-cycle is born (Note: $22 < a_3 < 23$). For $22 < a < a_3$, what do orbits of the map look like ?

2. (a) Compute *Lyapunov* exponents for the Baker's Map
$$x' = \mu x \ (\text{mod}1), \qquad 0 \leq x < 1$$
in the cases (i) $\mu = 2$ and (ii) $\mu = 3$.

   (b) Use Maple to compute the *Lyapunov* exponent for the orbit of the logistic map
$$x' = 3.9x(1 - x)$$
starting at $x_0 = 0.14285$

   (c) Use Maple to compute the *Lyapunov* exponent for the orbit of the *Ricker* map
$$x' = 30xe^{-x}$$
starting at $x_0 = 0.30969$

3. Construct an *OGY* controller for the *Ricker* map
$$x' = axe^{-x}, \qquad a_0 = 30$$
(the map is chaotic for the nominal parameter value $a_0 = 30$) which will enable the system to converge to the non zero fixed point in a superstable fashion and which will kick in whenever an orbit is within a distance no more than 0.05 from the fixed point (this specifies the "active region").
Repeat the exercise if you are told that the maximum allowable parameter variation is 2%. What is the size of the active region in this case?