MS4018 Problem Sheet 8

1. Consider the *Lorenz* system

$$\dot{x} = \sigma(y - x), \qquad \dot{y} = (\rho - z)x - y, \qquad \dot{z} = xy - bz$$

where $x(t), y(t)$ and $z(t)$ are variables which have a meteorological interpretation and σ, ρ and $b > 0$ are parameters. In investigating the system which is a toy model of a meteorological system, *Lorenz* "discovered" chaos using the parameter values $\sigma = 10, \rho = 28$ and $b = 8/3$.

   (a) Find the fixed points and determine their stability.

   (b) Using the accompanying Maple 3-d chaotic flows sheet, investigate the phenomenon of "sensitivity to initial conditions".

   (c) What happens as ρ is reduced? Identify the bifurcation which occurs. Does the system remain chaotic?

2. Consider the *Rössler* system

$$\dot{x} = -y - z, \qquad \dot{y} = x + ay, \qquad \dot{z} = b + z(x - c)$$

where $x(t)$, $y(t)$ and $z(t)$ are variables which have a chemical interpretation and a, b and $b > 0$ are parameters. *R össler* was interested in creating a chaotic system which mimicked a chemical reaction and had fewer nonlinearities than the *Lorenz* system. The original parameter values used were $a = b = 0.2$ and $c = 5.7$

   (a) Find the fixed points and determine their stability.

   (b) Using the accompanying Maple 3-d chaotic flows sheet, investigate the phenomenon of "sensitivity to initial conditions".

   (c) Fix b and c as above, and let a vary from -0.4 to $+0.4$. What dynamical features are seen?