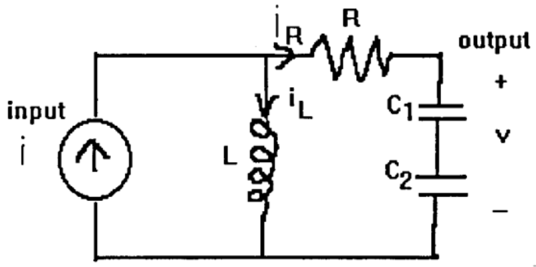
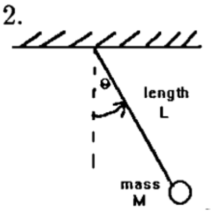
1. Find the system order, choose state variables and write the corresponding state model for the electrical system shown on right. Is the model linear ?



$$i = i_L + i_R, \quad L\frac{di_L}{dt} = Ri_R + v, \quad i_R = C_1\frac{dv_{C_1}}{dt} = C_2\frac{dv_{C_2}}{dt}$$

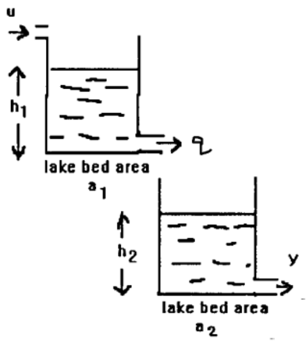
2.



$$m\frac{d^2}{dt^2}L\theta = -mg\sin\theta$$

Find the system order, choose state variables and write the state model for the pendulum shown on left. Linearise the system equations under the assumption that θ remains "small".

3. A two lake resevoir system is modelled as on right. The input flow rate ($u(t)$), output flow rate ($y(t)$) and inter-lake flow rate ($q(t)$) are measured in cubic metres per second. It may be assumed that the volume of water in each lake ($V_1(t)$ and $V_2(t)$ respectively) is directly proportional to the height of water in that lake ($h_1(t)$ and $h_2(t)$ respectively).



$$\dot{V_1} = u - q, \quad \dot{V_2} = q - y, \quad q = c_1\sqrt{h_1} \quad y = c_2\sqrt{h_2}$$

where c_1 and c_2 are constants.

The control objective is to maintain a fixed height of water in the lower lake, so as to give a constant pressure outflow.

(a) Find the system order, choose state variables and write down the state model.

(b) If the water in the lower lake is to be held at a height of H metres, find the input flow rate that, in steady state, will achieve this.

(c) Find the linearised model that describes how small variations, in the nominal steady state conditions of part (b), behave.