Review of Vectors

- **Scalar**: an entity or property of a physical system which can be represented by a single real number, e.g. temperature of a body, mass of a body.
- **Vector**: an entity with both a magnitude and a direction (and requiring three real numbers to fully specify it), e.g. the velocity of a car.
- **Scalar(dot, inner) product** of two vectors $\mathbf{a}$, $\mathbf{b}$:

  $$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta,$$

  where θ is the angle between the the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Alternatively

  $$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3,$$

  where the vectors in cartesian co-ordinates are $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$.
- **Cross(vector) product** of two vectors $\mathbf{a}$, $\mathbf{b}$:

  $$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta \, \mathbf{p},$$

  where $0 \le \theta \le \pi$ is the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ and $\mathbf{p}$ is a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a}, \mathbf{b}, \mathbf{p}$ form a right-handed triple. Alternatively

  $$\begin{aligned} \mathbf{a} \times \mathbf{b} &= \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \\ &= (a_2 b_3 - a_3 b_2)\mathbf{i} + (a_3 b_1 - a_1 b_3)\mathbf{j} + (a_1 b_2 - a_2 b_1)\mathbf{k} \\ &= (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1). \end{aligned}$$

- **Triple scalar product**

  $$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c} = \det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

- **Determinants**

  $$2 \times 2: \quad \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - b_1 a_2$$

  $$\begin{aligned} 3 \times 3: \quad \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} &= a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \\ &= a_1(b_2 c_3 - c_2 b_3) - a_2(b_1 c_3 - c_1 b_3) + a_3(b_1 c_2 - c_1 b_2) \end{aligned}$$

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