

SPRING 2007 - SOLUTIONS (PARTIAL) (1)

Qu1 - same as 2005 exam.

Qu2 - " " " "

Qu3 - " " " "

Qu4 - only difference: part (b):

$$u_{xx} + \alpha u_{xy} + 2u = 0 \quad \alpha > 0.$$

$$A=1, B=\frac{1}{2}\alpha, C=0$$

$$\therefore B^2 - AC = \frac{1}{4}\alpha^2 > 0$$

\Rightarrow hyperbolic
 \Rightarrow 2 characteristics

$$\therefore \frac{dy}{dx} = \frac{\frac{1}{2}\alpha \pm \sqrt{\frac{1}{4}\alpha^2}}{1}$$

$$\therefore \frac{dy}{dx} = 0$$

$$\Rightarrow y = c$$



$$\text{or } \frac{dy}{dx} = \alpha$$

$$\text{or } y = \frac{1}{2}\alpha x^2 + c$$

or



Qu 5 a

$$u_{tt} = c^2 u_{xx}$$

$$u(0, t) = u(l, t) = 0$$

$$u(x, 0) = f(x)$$

$$u_t(x, 0) = g(x)$$

$$\text{Let } u(x, t) = F(x)G(t)$$

$$\therefore u_{tt} = F\ddot{G} \quad \& \quad u_{xx} = F''G$$

$$\text{So } \frac{\ddot{G}}{c^2 G} = \frac{F''}{F} = k, \quad k \text{ const.}$$

$$\text{Solve } F''(x) - kF(x) = 0$$

$$\text{BCs } \Rightarrow F(0) = F(l) = 0.$$

only non-trivial solution

$$F_n(x) = D_n \sin \frac{n\pi x}{l}$$

(see Qu 5, 2005 exam).

is for $k = -p^2 < 0$

WOULD NEED MORE
DETAIL TO GET
FULL MARKS.
i.e. $k=0$ & $k > 0$ cases also.

Qu 5 cont

(2)

$$k = -p^2 = -\frac{n^2 \pi^2}{L^2}$$

$$\therefore \text{Solve } \ddot{G} + \lambda_n^2 G = 0 \quad \lambda_n = \frac{n\pi c}{L}$$

General solⁿ

$$G_n(t) = \bar{A}_n \cos \lambda_n t + \bar{B}_n \sin \lambda_n t$$

$$\text{Thus } u_n = F_n G_n$$

$$\Rightarrow u(x, t) = \sum_{n=1}^{\infty} F_n(x) G_n(t)$$

$$= \sum_{n=1}^{\infty} (A_n \cos \lambda_n t + B_n \sin \lambda_n t) \sin \frac{n\pi x}{L}$$

$$\text{i.e. } u(x, 0) = f(x)$$

$$\therefore f(x) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{L} \Rightarrow A_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$$

$$u_t(x, t) = \sum_{n=1}^{\infty} (-A_n \lambda_n \sin \lambda_n t + \lambda_n B_n \cos \lambda_n t) \sin \frac{n\pi x}{L}$$

$$\text{i.e. } u_t(x, 0) = g(x)$$

$$\therefore g(x) = \sum_{n=1}^{\infty} \lambda_n B_n \sin \frac{n\pi x}{L} \Rightarrow B_n = \frac{2}{\lambda_n L} \int_0^L g(x) \sin \frac{n\pi x}{L} dx$$

(b) same as 2005 exam.

Qu 6

Parts (a) - (b) BOOK WORK

Part (c), would not ask to calculate iterations, just set up system of linear eqns (like Qu 6(d)).